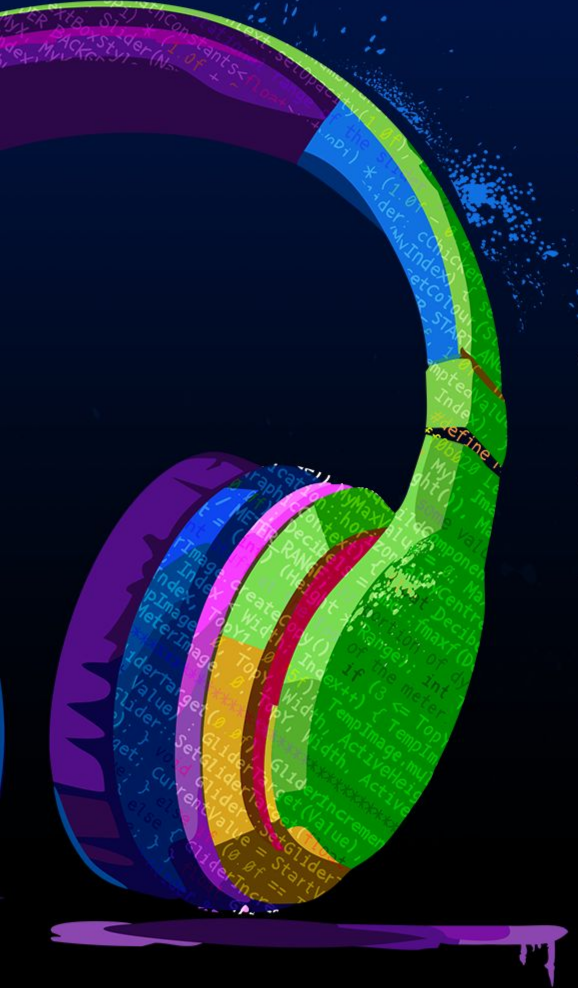


ADC24 Bristol

THE INTERSECTION OF ANALOG AND DIGITAL AUDIO PROCESSING

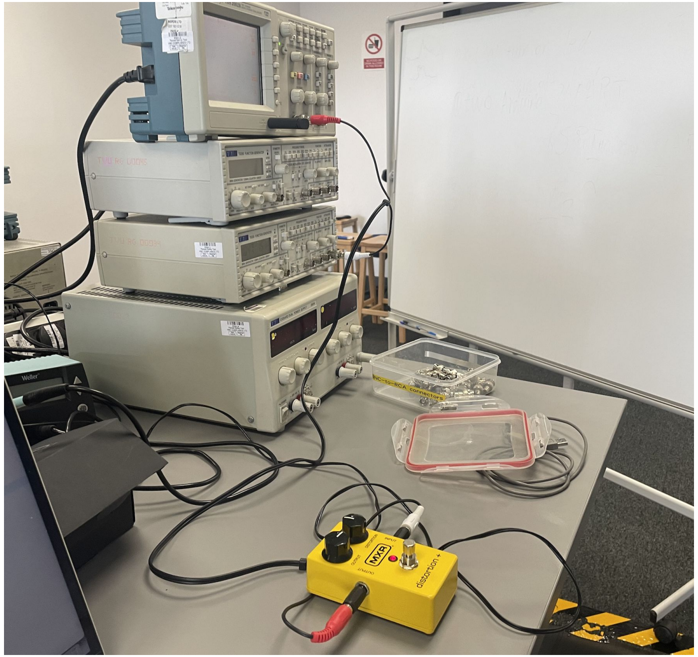
AN INTRODUCTORY GUIDE TO VIRTUAL ANALOG MODELLING

SOHYUN IM

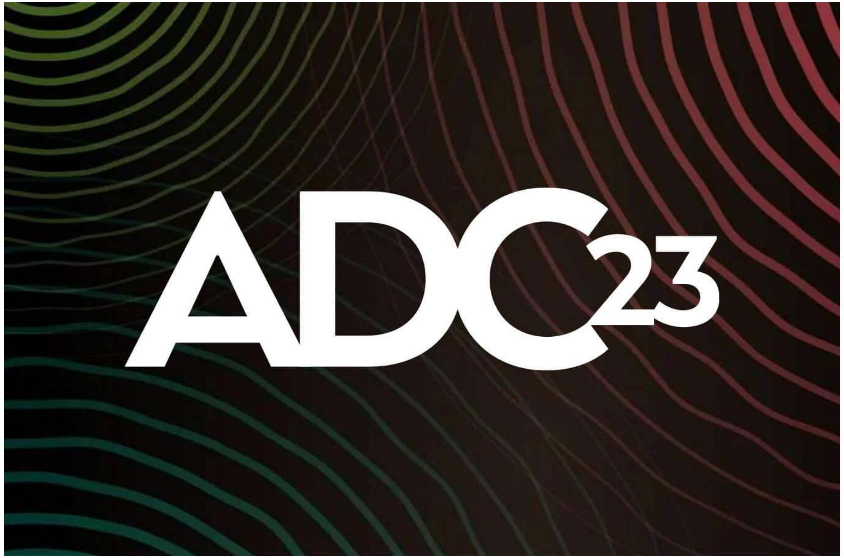
About me

- **2021-2024**: BSc in Sound Engineering, University of West London
- **2024-present**: Master’s student in Sound and Music Computing, Queen Mary University of London
- Amatuer pianist
- ADC Diversity Scholarship recipient, 2022 and 2023.
- Giving my first talk at ADC24
- Participated in Dynamic Cast meetup

A Little Background on the Project Behind This Talk



Electronics Lab at University of West London



Audio Developer Conference 2023

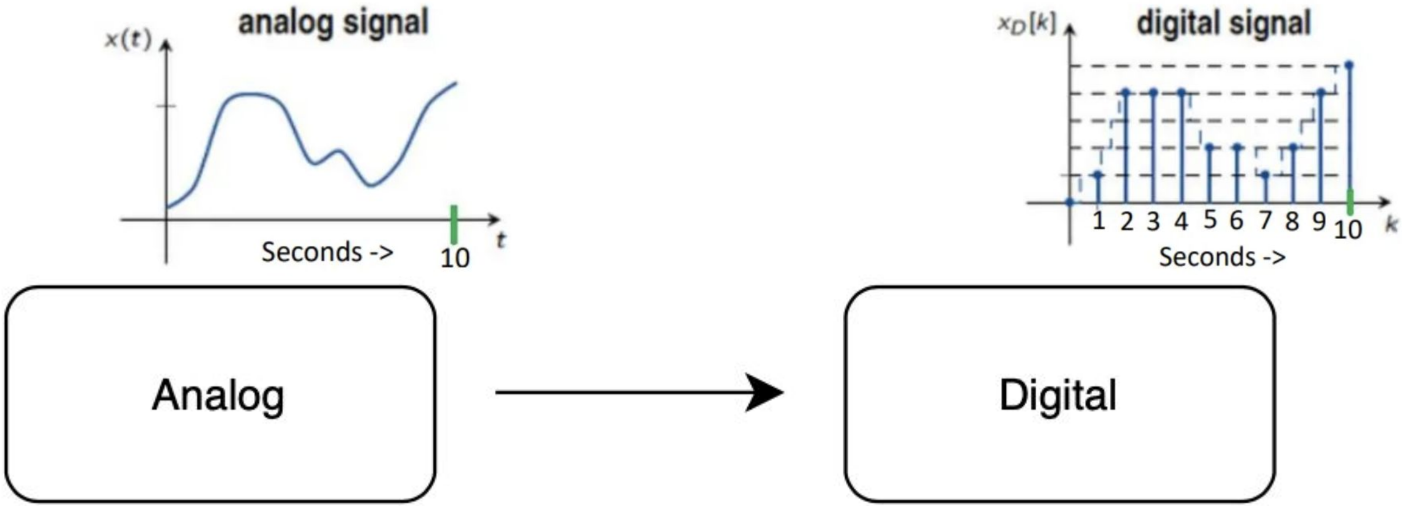
What I Aim to Share

- Simple yet concrete examples for newcomers to Virtual Analog Modelling
- Clarifying confusing terminology in the field
- Sharing the fun of understanding circuits

Repository

https://github.com/Sohyun-Im/adctalk24

What is Virtual Analog (VA) Modelling?

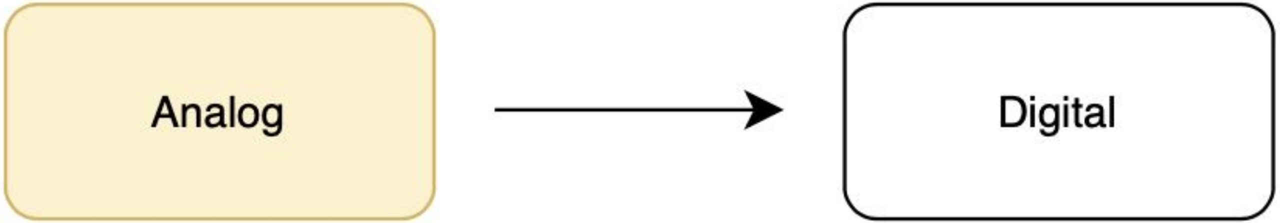


A system that processes continuous input/output signals that vary over time

A system processes discrete input/out signals at specific intervals over time.

https://medium.com/@vitieeesps/analog-and-digital-signals-two-sides-of-the-same-coin-dc06b0d242c4

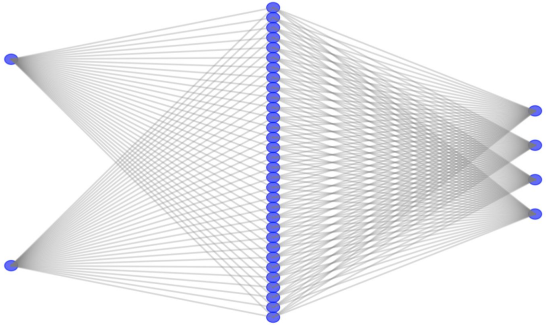
What is Virtual Analog (VA) Modelling?



What is Virtual Analog (VA) Modelling?

```
for n = 1:N

    % input-sample
    Vin = u(n,1);

    % transfer function to obtain the output-sample
    Vout = b0 * Vin + b1 * x1 + b2 * x2;

    % equations required for state updates equations
    Vx = (1/Gh)*Vout + ((R2*R4)/(Gb*Gh*R5))*x2;
    VR1 = (Vin-Vx+(R3*x1))/Ga;

    VR5 = (Vx-(R2*x2))/Gb;
    VR2 = (R2/R5)*VR5 + (R2*x2);

    % state update equations for C1 and C2
    x1 = (2/R1)*VR1 - x1;
    x2 = (2/R2)*VR2 - x2;

    % y = output signal vector
    y(n,1) = Vout;

end
```

Analog → Digital



What is Virtual Analog (VA) Modelling?

```
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    % equations required for state updates equations
    Vx = (1/Gh)*Vout + ((R2*R4)/(Gb*Gh*R5))*x2;
    VR1 = (Vin-Vx+(R3*x1))/Ga;

    VR5 = (Vx-(R2*x2))/Gb;
    VR2 = (R2/R5)*VR5 + (R2*x2);

    % state update equations for C1 and C2
    x1 = (2/R1)*VR1 - x1;
    x2 = (2/R2)*VR2 - x2;

    % y = output signal vector
    y(n,1) = Vout;

end
```
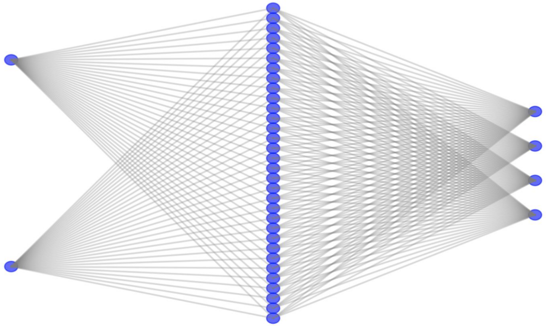

Creating a digital signal processor that mimics the electrical behaviour of a reference circuit



VA Modelling vs Physical Modelling

Physical modelling

"… emulates the behaviour of sound-producing objects in the real world."

e.g. acoustic instruments: piano, strings, etc.

Modartt - Pianoteq

https://babyaud.io/blog/what-is-physical-modeling#:~:text=Physical%20modeling%20is%20the%20term,can%20explore%20with%20physical%20modeling.

Introduction to the Task

VA modelling of the **MXR *Distortion+*** guitar pedal based on circuit analysis methods

- Circuit analysis methods for analysing individual component and the complete circuit.
- Modelling by sub-circuit
- Implementation of Digital Signal Processing (DSP) using Python in Google Colab.
- Listening test: Comparison between VA-modelled DSP and the real pedal.

How Does It Sound Like?

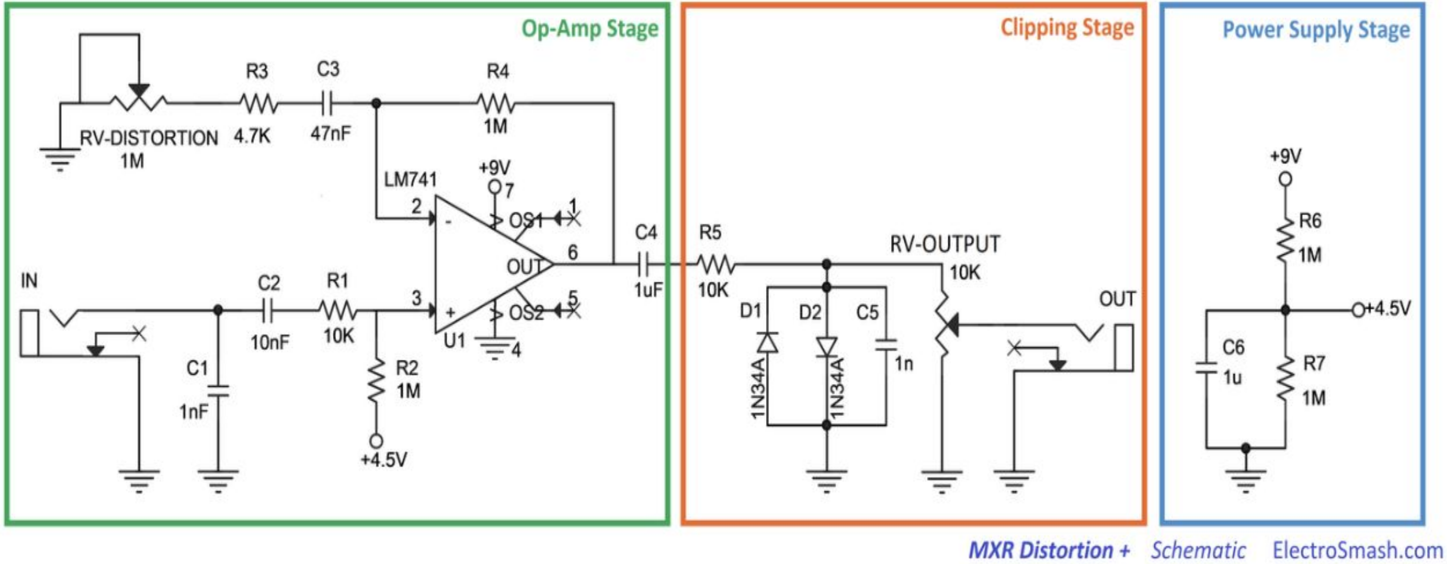
MXR *Distortion+*

- **1970s** MXR Innovations
- **1987** Acquired by Jim Dunlop

- Randy Rhoads (w/ Ozzy Osbourne)
  “Crazy Train”, “Mr. Crowley” (1980)

- Jerry Garcia in Grateful Dead
  “Shakedown Street” (1978)

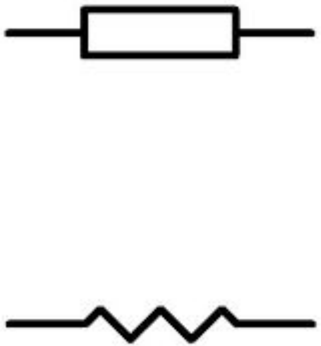
Overview of the Schematics



Some Prerequisite Knowledge (1) Voltage & Current

- Voltage (potential difference)
  - A pressure of electricity, which pushes electrons from a point of higher potential to a point of lower potential.
  - measured in volts (V)
- Current
  - The amount of electron flow in a circuit.
  - measured in amperes (A)

Some Prerequisite Knowledge (2) Components - Resistors

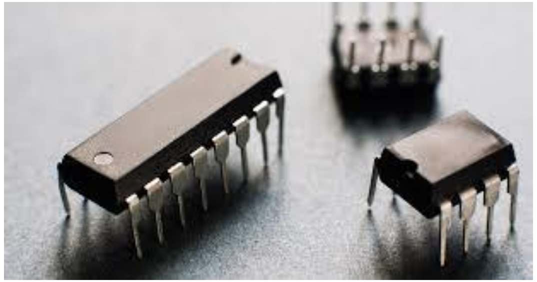


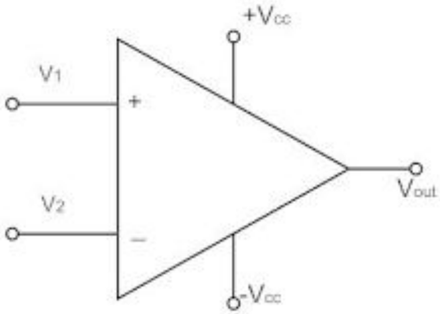
- Resistors limit the amount of current and divide voltage in a circuit, acting like a bottleneck
- Measured in Ohms(Ω)

Some Prerequisite Knowledge (2) Components - Capacitors

- Capacitors can store/release electrical energy.
- The current through a capacitor and the voltage across it keep changing.
- Measured in Farads (F)

Some Prerequisite Knowledge (2) Components - Operational Amplifiers (Op-amps)



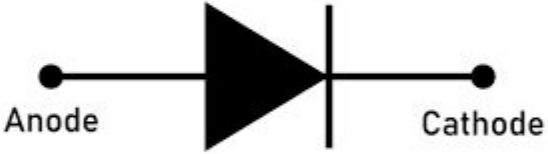


Op-Amp Symbol

- High-gain voltage amplifier
- Two input terminals
  - inverting input
  - non-inverting input
- Key rules
  - If there's a feedback loop in a circuit, the voltages at the two input terminals are always equal.
  - No current flows into the two input terminals.

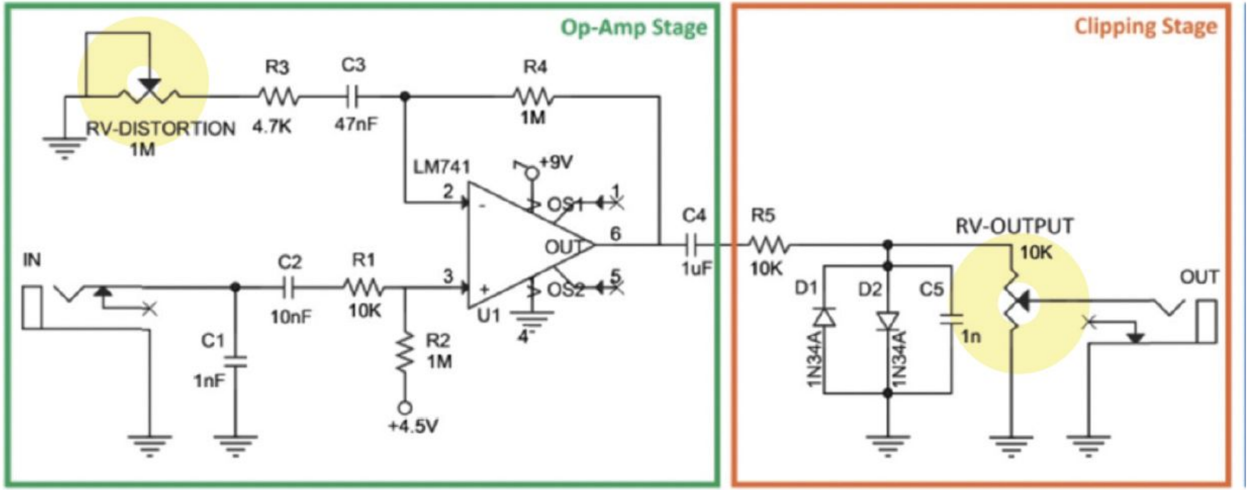
Some Prerequisite Knowledge (2) Components - Diodes

- Current flows in one direction from the Anode to the Cathode, blocking reverse flow.

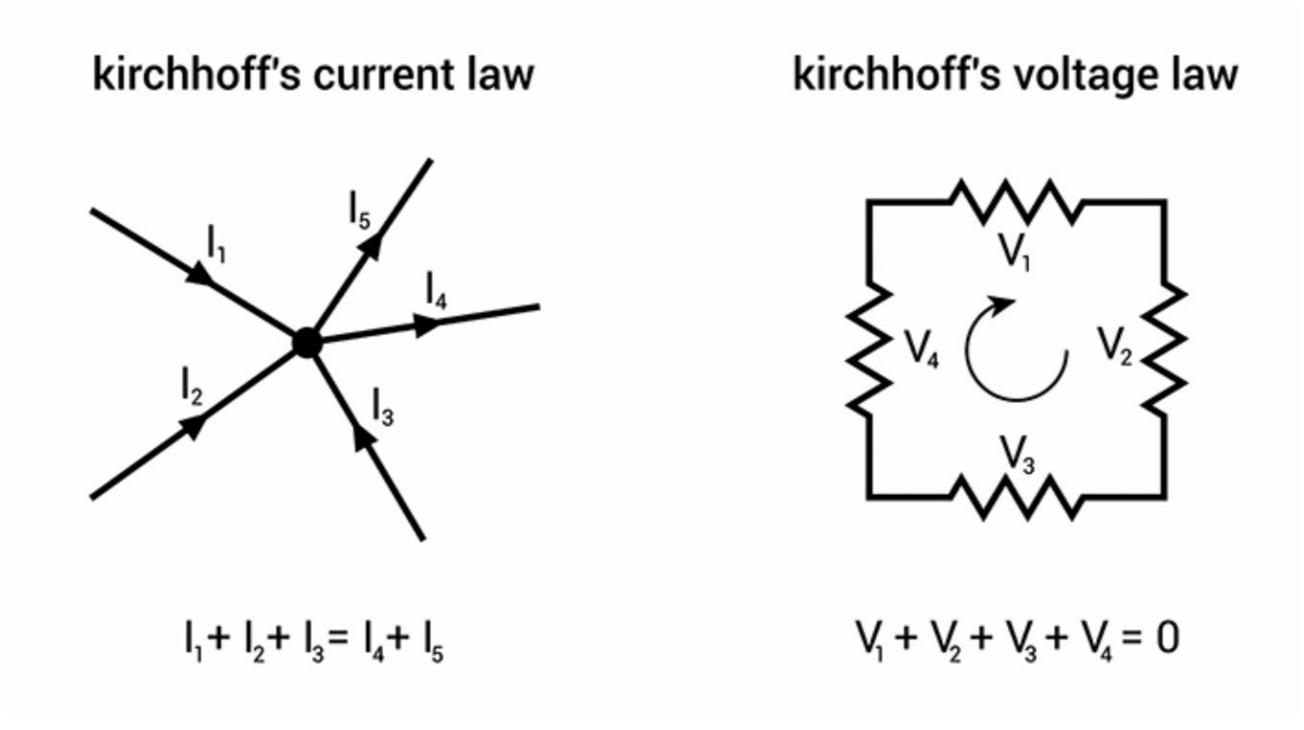


Some Prerequisite Knowledge (2) Components - Potentiometers (Variable Resistors)

- Potentiometers adjust resistance within a given range(Ω) to control voltage and current.



Some Prerequisite Knowledge (3) Kirchhoff's Law



https://www.student-circuit.com/learning/the-importance-of-kirchhoffs-laws-in-electrical-engineering/

Some Prerequisite Knowledge (4) Calculus

Differentiation and Derivative

- rate of change at a specific moment
- slope

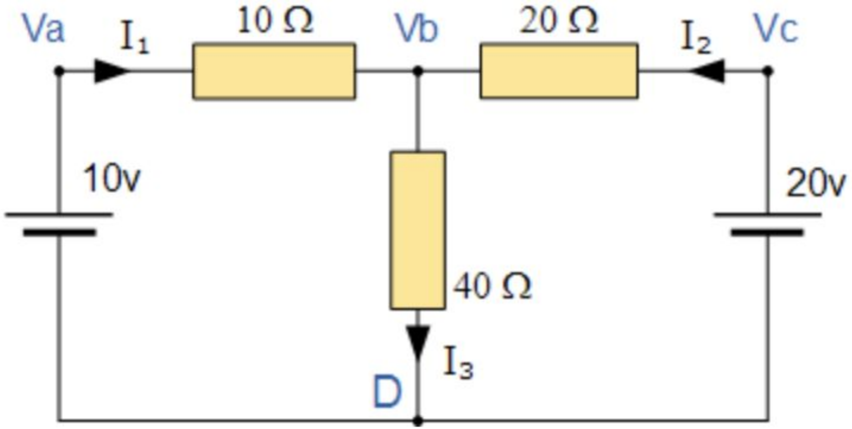
Integration and Integral

- calculating the accumulated total.
- calculating the area under a function’s curve

Circuit Analysis Methods

Circuit Analysis Method 1. Nodal Voltage Analysis (NVA)

- Label a node in the circuit with unknown voltage values
- Formulate equations using KCL or KVL for these nodes



$$\frac{(V_a - V_b)}{10} + \frac{(V_c - V_b)}{20} = \frac{V_b}{40}$$

$$\boldsymbol{V = I \times R}$$

- ***V*** is the voltage (V)
- ***I*** is the current (A)
- ***R*** is the resistance (Ω)

https://www.electronics-tutorials.ws/dccircuits/dcp_6.html

Circuit Analysis Method 1. Nodal Voltage Analysis (NVA)

- Using NVA, we can further find out a mathematical expression that defines the relationship between input and output, known as the transfer function.
- This transfer function allows us to apply a discrete input to a system originally designed for continuous analog signals.

Circuit Analysis Method 2: Ohm's Law

- Ohm's Law expresses the relationship between voltage, current, and resistance in a circuit

$$V = I \times R$$

- ***V*** is the voltage (V)
- ***I*** is the current (A)
- ***R*** is the resistance (Ω)

Circuit Analysis Method 3: What about Capacitors?

- Finding the voltage across and the current through a capacitor is not as simple as with a resistor

- because a capacitor is dynamic component with time-varying voltage and current

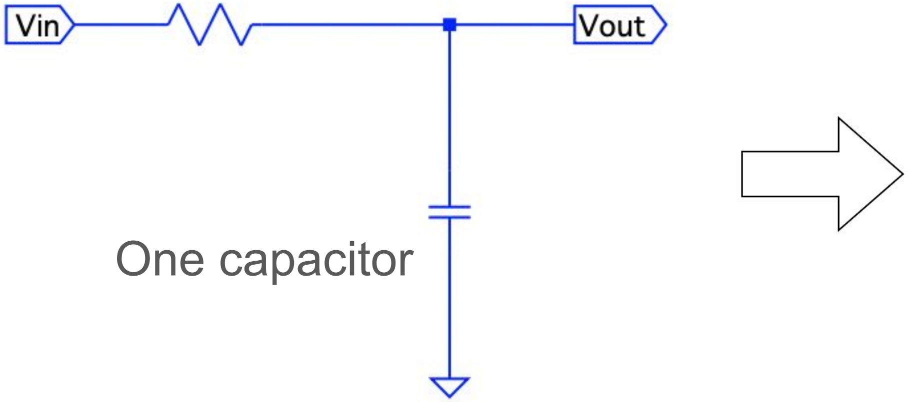
3.1 Capacitance and current

The voltage across and the current through the capacitor over time

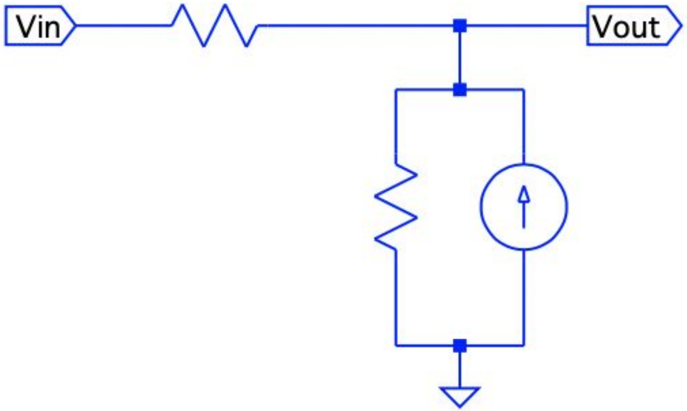
$$V_c = \frac{1}{C}\int I_c dt, \qquad I_c = C\frac{dV_C}{dt}$$

(1) (2)

3.2 Discrete Kirchhoff (DK) Substitution on Capacitors

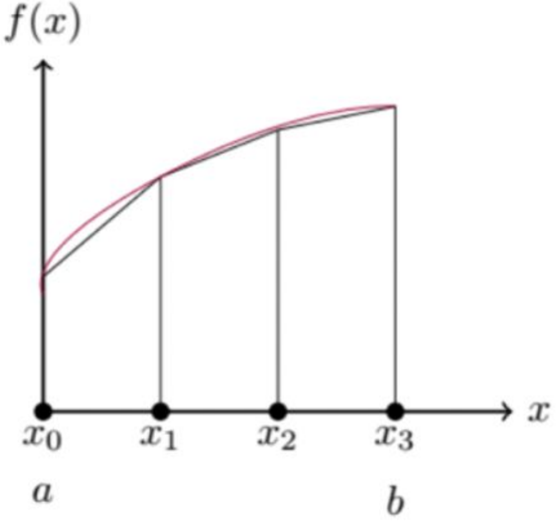


One capacitor



One resistor and one reversed current source in a parallel configuration

3.3.1 The principles behind the DK substitution for capacitors



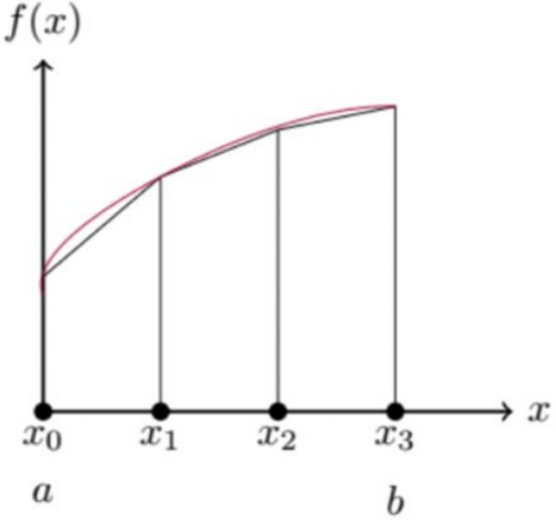
With respect to the discrete time point x2:

- The voltage across the capacitor is the area under the function's curve up to the x2.
- The current flowing through the capacitor is the rate of change of voltage multiplied by the capacitance.

(1) (2)

$$V_c = \frac{1}{C}\int I_c dt, \quad I_c = C\frac{dV_C}{dt}$$

3.3.2 The principles behind the dk substitution for capacitors



(1) $$\int f(x_3)dt - \int f(x_2)dt = \left(\frac{f(x_2) + f(x_3)}{2}\right) \cdot Ts$$

Equation (1) represents the current flowing through the capacitor at the discrete time point x2 as the difference between the accumulated voltage up to x3 and that up to x2.

(2) $$V[n] - V[n-1] = \frac{i[n] + i[n-1]}{2} \cdot T_s \cdot \frac{1}{C}$$

Equation (2) substitutes (1) into the discrete-time voltage change equation, reformulating it for the discrete signals n and n−1.

3.3.3 The principles behind the dk substitution for capacitors

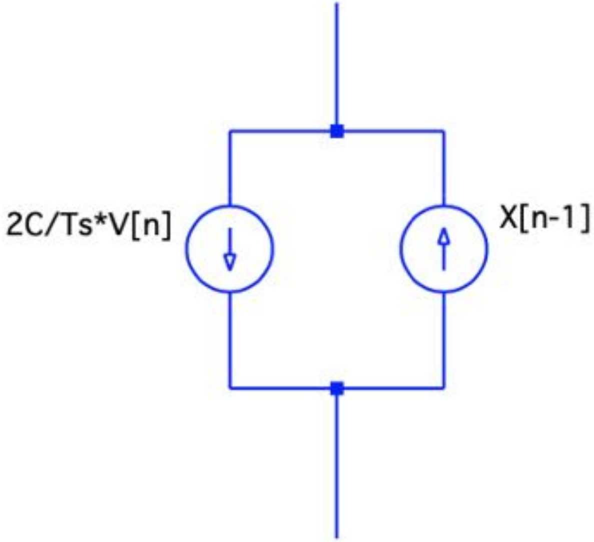
(1) $$V[n] - V[n-1] = \frac{i[n] + i[n-1]}{2} \cdot T_s \cdot \frac{1}{C}$$

(2) $$i[n] = \frac{2C}{Ts} \cdot V[n] - \frac{2C}{Ts} \cdot V[n-1] - i[n-1]$$

(2.1) $$X[n-1] = \frac{2C}{Ts} \cdot V[n-1] + i[n-1]$$

state value

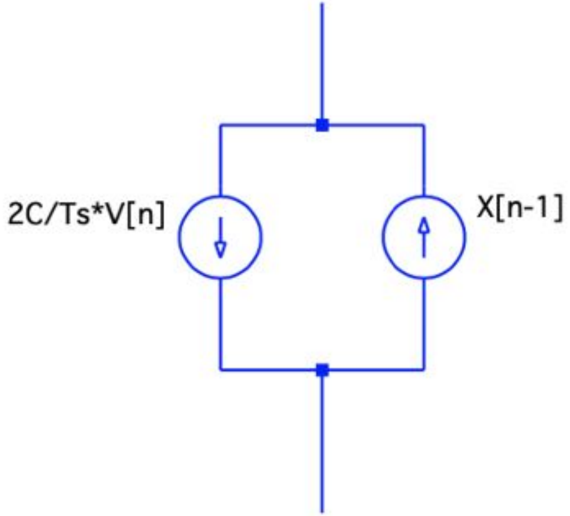
(3) $$i[n] = \frac{2C}{Ts} \cdot V[n] - X[n-1]$$



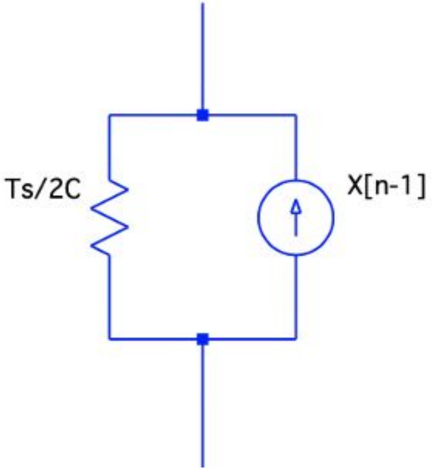
3.3.4 The principles behind the dk substitution for capacitors

(1) $$i[n] = \frac{2C}{Ts} \cdot V[n] - X[n-1]$$

(2) $$i = V[n] \cdot \frac{2C}{Ts}$$



(a)

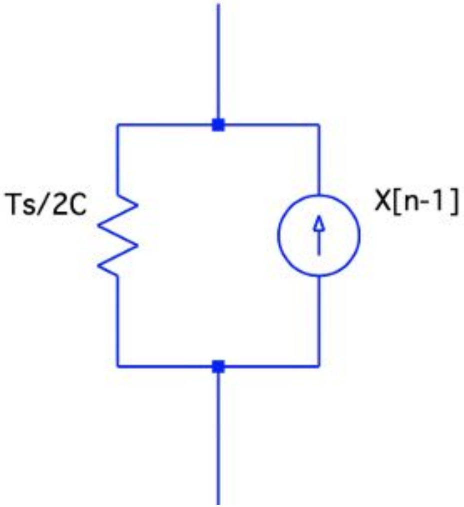


(b)

(3) $$\frac{1}{R} = \frac{2C}{Ts}$$

(4) $$R = \frac{Ts}{2C}$$

3.4 state update equation



(1) $$X[n-1] = \frac{2C}{Ts} \cdot V[n-1] + i[n-1]$$

(2) $$X[n] = \frac{2C}{Ts} \cdot V[n] + i[n]$$

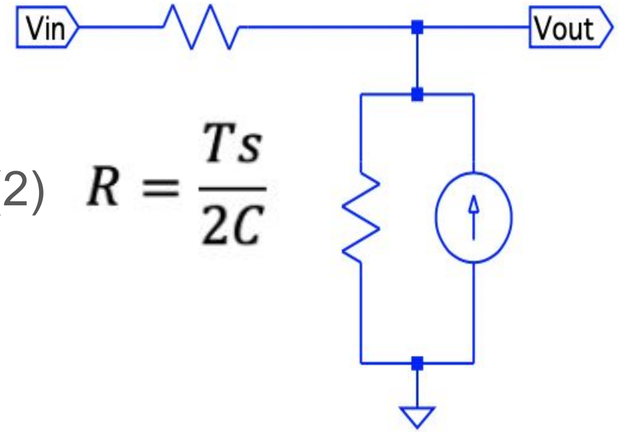
(3) $$i[n] = \frac{1}{R} \cdot V[n] - X[n-1]$$

(4) $$\therefore \; X[n] = \frac{2}{R} \cdot V[n] - X[n-1]$$

Method 3 Takeaways: Circuit Analysis with Capacitors

(1) $$I_c = C\frac{dV_C}{dt} \Rightarrow i[n] = \frac{2C}{Ts}\cdot V[n] - X[n-1]$$

over time *t* — by discrete sample [n]



(2) $$R = \frac{Ts}{2C}$$

(3) State value

$$X[n-1] = \frac{2C}{Ts}\cdot V[n-1] + i[n-1]$$

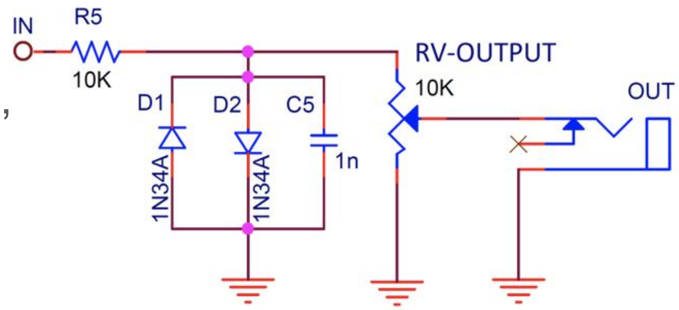
(4) State-update equation

$$X[n] = \frac{2}{R}\cdot V[n] - X[n-1]$$

Method 4. About Diodes

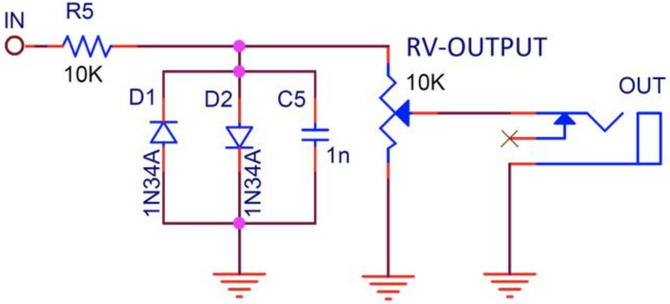
- Back-to-back configuration: One diode conducts current when the signal flows in the positive direction, while the other diode conducts when the signal flows in the negative direction.

- Clipping voltage (0.7V): When the signal is below 0.7V, it passes through unaffected. However, when the signal exceeds 0.7V, current flows through the diode, limiting the signal at the 0.7V threshold and creating a clipping effect.



This results in **a distorted sound** in the pedal, where the fundamental pitch is preserved while additional harmonics are introduced, giving a richer and more aggressive tone.

Method 4.1 Shockley Diode Equations

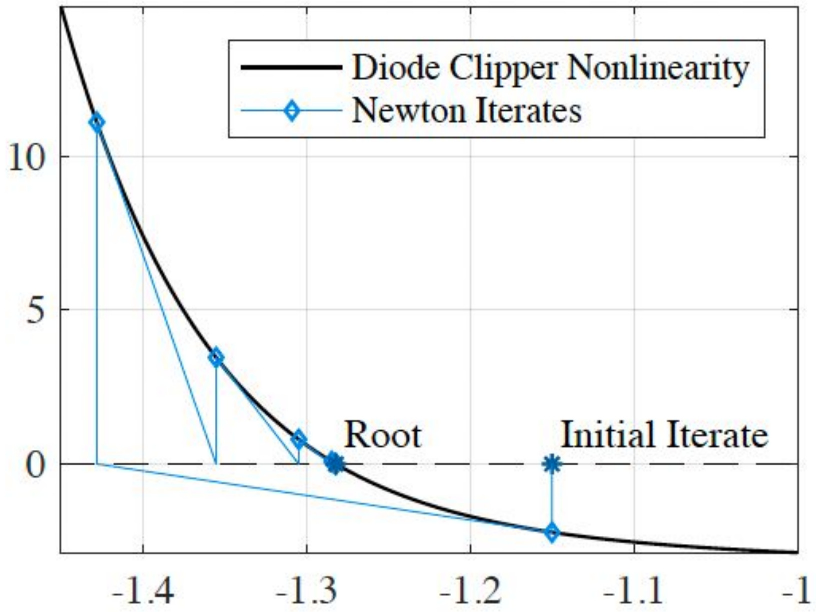


(1) $$i_d = I_s * [e^{(\frac{V_d}{V_T \cdot \eta})} - 1]$$

(2) $$i_{d\_pair} = 2 \cdot I_s \cdot \sinh\left(\frac{V_d}{\eta \cdot V_T}\right)$$

```
# Germanium diode params
Is = 10e-6  # saturation current
Vt = 0.026  # thermal voltage
eta = 2  # emission coefficient
```

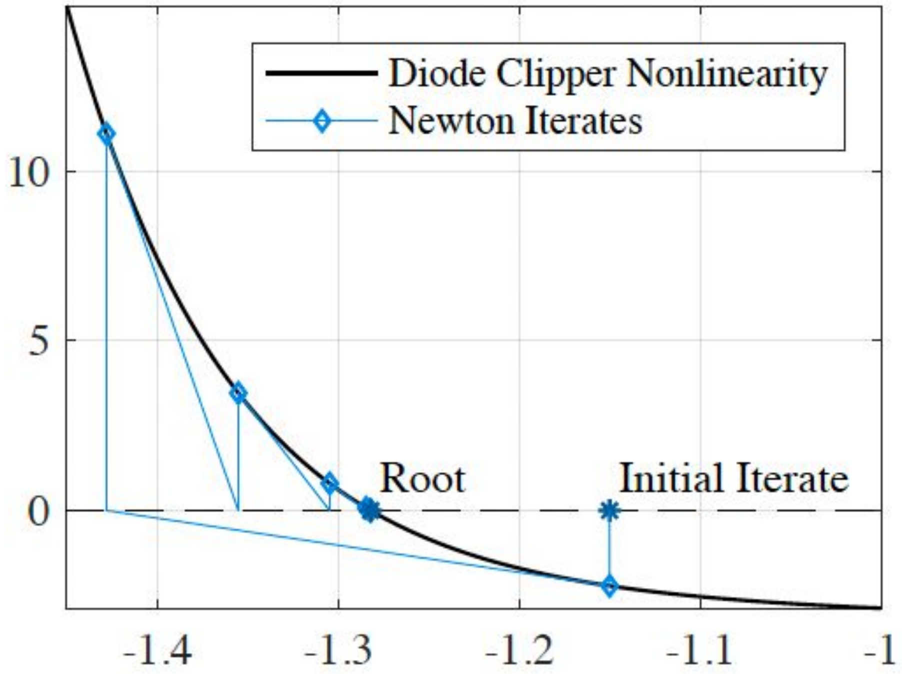
Method 4.2 Newton-Raphson Method



- To solve for Vd, the equation cannot be directly solved, because Vd is inside the sinh().
- To address this, the equation is rearranged so that all terms are on one side, and call this function f(Vd).
- The goal is to find the value of Vd that makes this function f(Vd) = 0.
- Newton Raphson method starts with arbitrary point as a initial guess.
- Then the slope f'(Vd) of the functon at the initial point is calculated.
- Check where this slope intersects the x-axis,
- Check the function f(Vd)'s value. If it's not 0, repeat this process.

Holmes, B. (2019). Guitar Effects-Pedal Emulation and Identification. Queen's University Belfast.
https://pureadmin.qub.ac.uk/ws/portalfiles/portal/169350312/main.pdf

Method 4.2 Newton-Raphson State Update Equation



$$f'(x_0) = \frac{f(x_0)}{x_0 - x_1} \qquad (1)$$

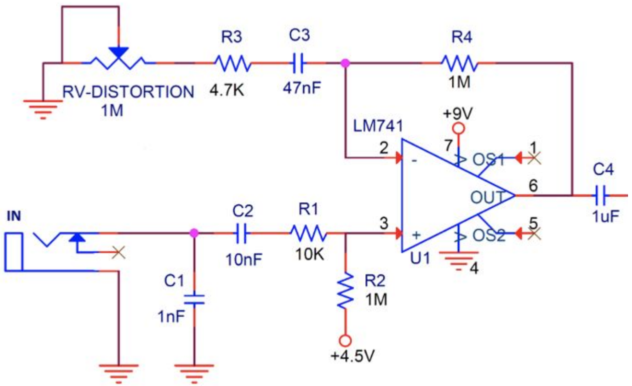
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad (2)$$

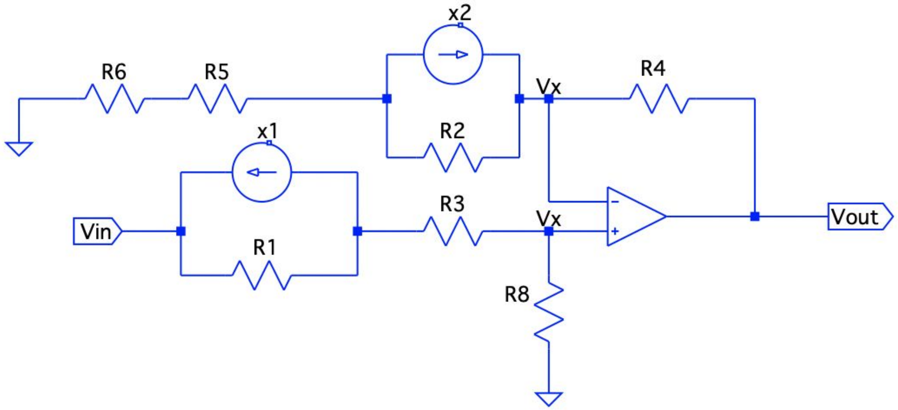
'x0' is the initial guess to find a root of the function, 'x1' is the second guess

Holmes, B. (2019). Guitar Effects-Pedal Emulation and Identification. Queen's University Belfast.
https://pureadmin.qub.ac.uk/ws/portalfiles/portal/169350312/main.pdf

Op-amp stage modelling

DK Substituted Circuit of Op-amp Stage



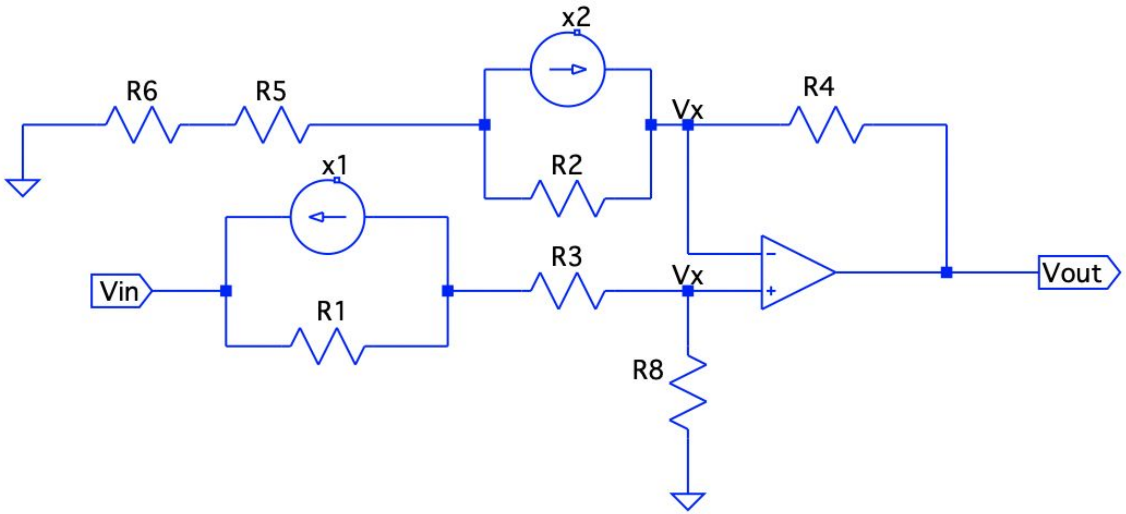


C1 and C4 are removed because those didn’t affect the signal

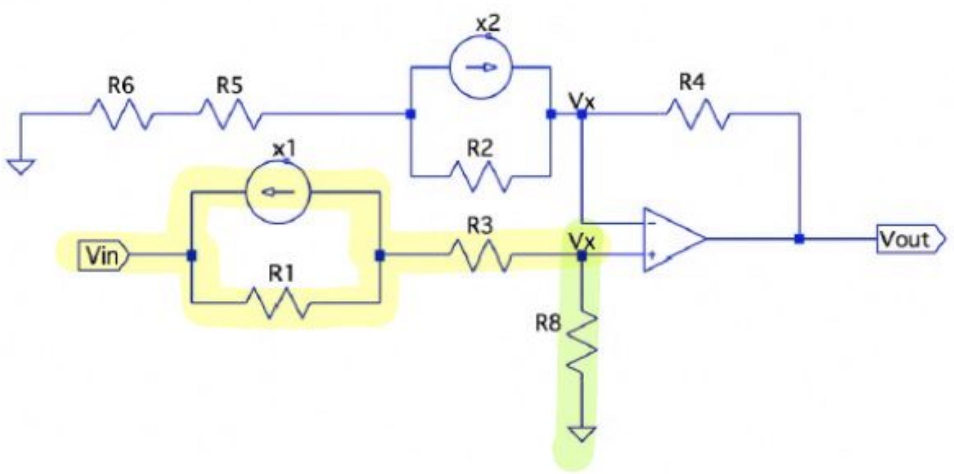
- Components are re-labeled
- R6 is Distortion potentiometer (knob)

What We Aim to Find?

- Set up two equations for Vx, then eliminate Vx.
- Derive the transfer function that represents the relationship between Vin and Vout.



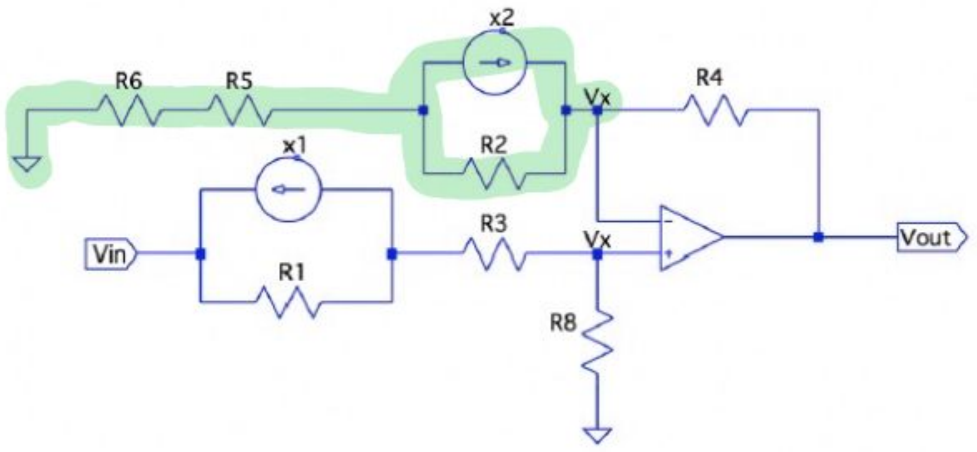
Process (1) Two branches of the input section



At the non-inverting input node Vx, using KCL, the following equation is derived based on the fact that the current flowing into Vx from Vin is equal to the current flowing out of Vs through R8

$$V_x = \frac{1}{G_a \cdot R1 \cdot G_x} \cdot V_{in} - \left(1 - \frac{R3}{G_a \cdot R1}\right)\left(\frac{x1}{G_x}\right)$$

Process (2) Potentiometer section

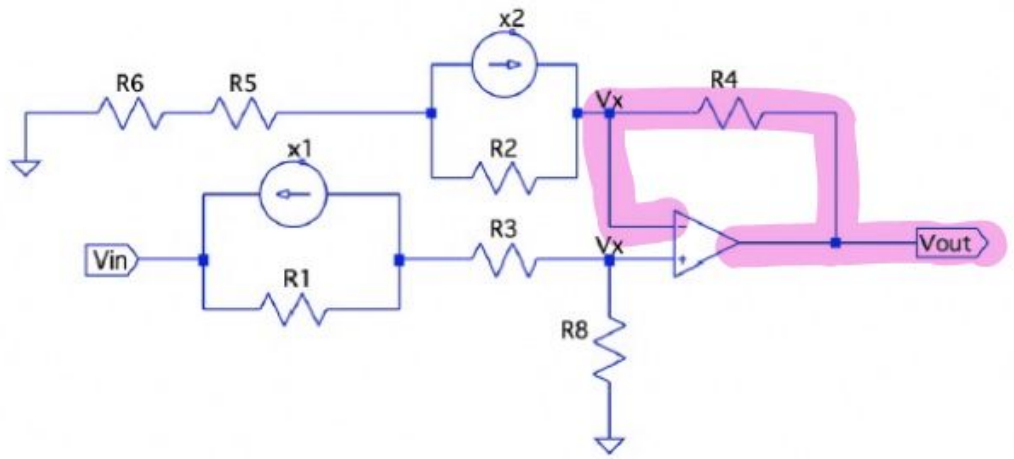


The current ic, exiting the node Vx, is calculated using the fact that the current is the same throughout the segment, along with the equations for the current flowing through the resistor and capacitor.

$$i_c = \frac{V_x - R2 \cdot x2}{G_b \cdot Rn}$$

Rn = R5 + R6

Process (3) Feedback loop section



Using KCL wrt the node Vx with the obtained current value from the previous process, find the equation for Vx expressed with Vout and the component values.

$$V_x = \frac{1}{G_h} \cdot V_{out} + \frac{R2 \cdot R4}{G_b \cdot Rn \cdot G_h} \cdot x2$$

Process (4) Transfer function

- The two equations for Vx obtained in Process (1) and Process (3) are formed a simultaneous equation.

- After Vx is eliminated, the transfer function of the Op-amp stage is obtained by solving the remained equation for Vout.

$$V_{out} = \frac{G_h}{G_a \cdot R1 \cdot G_x} \cdot V_{in} + \left(\frac{R3}{G_a \cdot R1} - 1\right)\left(\frac{G_h}{G_x}\right) \cdot x1 + \left(\frac{-R2 \cdot R4}{G_b \cdot Rn}\right) \cdot x2$$

Process (5) State-update equation

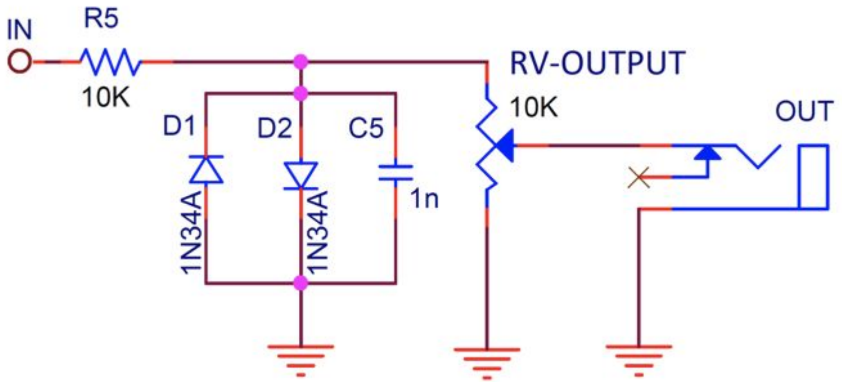
When the n+1th discrete signal is introduced, the state values for C1 and C2, will be updated through the state-update equations (1), (2).

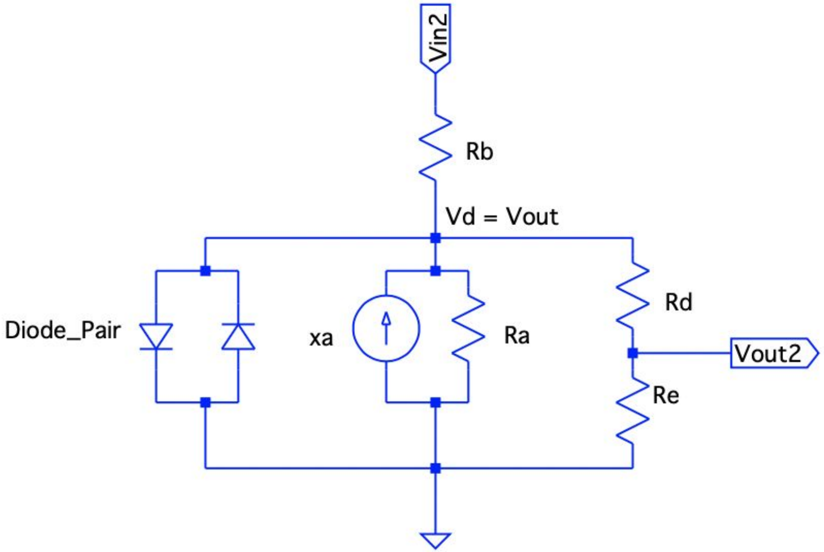
(1) $$x_1[n+1] = \frac{2}{R1} \cdot V_{R1} - x_1[n]$$

(2) $$x_2[n+1] = \frac{2}{R2} \cdot V_{R2} - x_2[n]$$

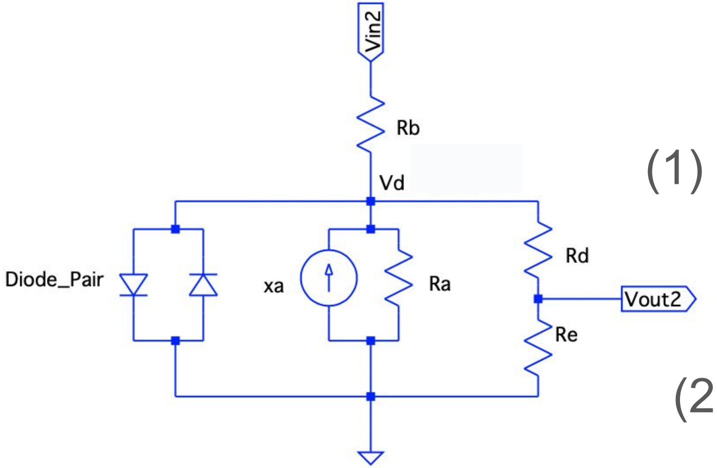
Clipping stage modelling

DK Substitution of Clipping Stage





DK Substitution of Clipping Stage



First, NVA for the node 'Vd' using KCL…

$$(1)\quad \frac{V_{in}2 - V_d}{Rb} = 2 \cdot Is \cdot \sinh\left(\frac{V_d}{\eta \cdot V_T}\right) + \frac{V_d}{Ra} - x_a + \frac{V_d - V_{out}2}{Rd}$$

Second, move all terms of the equation (1) to one side and express iit as a function f(Vd) in terms of Vd.

$$(2)\quad f(V_d) = \frac{V_d}{Rb} - \frac{V_{in}2}{Rb} + 2 \cdot Is \cdot \sinh\left(\frac{V_d}{\eta \cdot V_T}\right) + \frac{V_d}{Ra} - x_a + \frac{V_d}{Rd} - \frac{V_{out}2}{Rd}$$

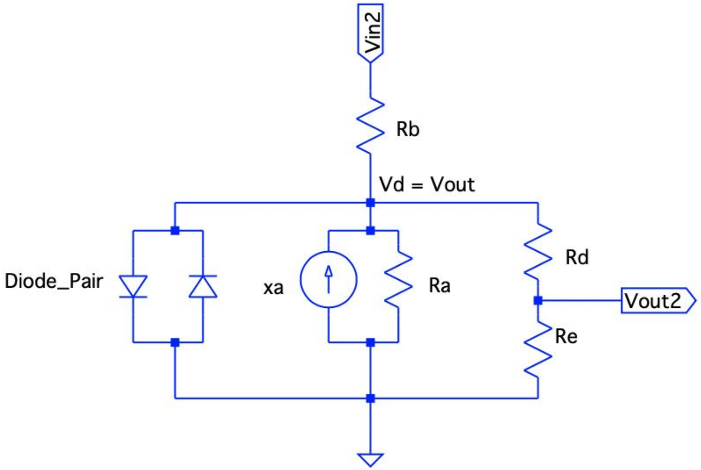
Third, calculate the derivative of the function f(Vd) to determine the next guess for the Newton-Raphson iteration

$$(3)\quad f'(V_d) = \frac{1}{Rb} + \frac{1}{Ra} + \frac{1}{Rd} + \frac{2 \cdot Is}{\eta \cdot V_T} \cdot \cosh\left(\frac{V_d}{\eta \cdot V_T}\right)$$

Newton-Raphson update equation

$$(4)\quad \mathrm{V_d} = \mathrm{V_d} - \frac{\mathrm{f(V_d)}}{\mathrm{f'(V_d)}}$$

Derive the Transfer Function for the Clipping Stage



By applying KCL,

$$\frac{V_d - V_{out}2}{R_d} = \frac{V_{out}2}{R_e}$$

By rearranging the equation wrt Vout2, the following transfer function for the clipping stage is derived.

$$V_{out}2 = \frac{V_d}{R_d \cdot G_g}$$

DSP Implementation with Python (CoLab)

Takeaways

- Definition of Virtual Analog Modeling
- Various Circuit Analysis Methods for Discretizing Systems
    - Nodal Voltage Analysis
    - Ohm's Law for Resistors
    - DK Substitution for Capacitors
    - Shockley Diode Equation and Newton-Raphson Method for Diodes
- DSP Implementation with Python
- Listening Test

Q&A